



Exclusive Reactions Involving Pions and Nucleons

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Nomenclature

Note: the units used in this work are such that $\hbar = c = 1$.

EM	electromagnetic
1	symbol for projectile particle in the reaction $1 + 2 \rightarrow \text{anything}$
2	symbol for target particle in the reaction $1 + 2 \rightarrow \text{anything}$
cm	symbol for center of momentum frame, where $\mathbf{p}_1 + \mathbf{p}_2 = 0$
lab	symbol for lab (target) frame, where $\mathbf{p}_2 = 0$
x^*	denotes a variable x in the cm frame
x_{lab}	denotes a variable x in the lab frame
x	denotes a variable x in the lab frame <i>or</i> denotes an invariant variable x
m_π	pion mass, GeV
m_p	proton mass, GeV
m_i	mass of particle i , GeV
T	particle kinetic energy, GeV
E	particle total energy, GeV
p	magnitude of 3-momentum, $p = \mathbf{p} $, GeV
	Note: not to be confused with symbol for proton. Context will clarify.
β	speed of cm frame (unitless)
γ	relativistic γ factor, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ (unitless)
$\frac{d^3\sigma}{dp^3/E}$	Lorentz invariant differential cross section, mb/GeV ²
$\frac{d\sigma}{dE}$	spectral distribution cross section, mb/GeV
σ	total cross section, mb
$\frac{d^2N}{dp_{ }dp_{\perp}}$	single particle distribution in the projectile-nucleon cm frame
\sqrt{s}	total energy in cm frame, $\sqrt{s} = E_{1\,cm} + E_{2\,cm} \equiv E_{cm}$, GeV
\mathbf{p}_π^{*2}	pion momentum in cm frame, GeV
$\mathbf{p}_{\pi\,max}^{*2}$	maximum pion momentum in cm frame, GeV
$\theta_{\pi\,max}$	maximum pion angle in lab frame
$p_{1\,lab}$	magnitude of 3-momentum of particle 1 in the lab frame for the reaction $1 + 2 \rightarrow 3 + 4$
	$p_{1\,lab} = \mathbf{p} _{1\,lab}$, GeV
$E_{1\,lab}$	total energy of particle 1 in the lab frame for the reaction $1 + 2 \rightarrow 3 + 4$
	$E_{1\,lab} = \sqrt{ \mathbf{p} _{1\,lab}^2 + m_1^2}$, GeV
p^μ	4-momentum vector, $p^\mu = (E, \mathbf{p})$, GeV
$(p^\mu)^2$	square of 4-momentum vector, $(p^\mu)^2 = E^2 - \mathbf{p}^2 = m^2$, GeV ²
p_i^μ	4-momentum vector of particle i , GeV

N	symbol for nucleon (either proton or neutron)
n, N^0	neutron
\bar{n}	antineutron
p, N^+	proton
	Note: not to be confused with magnitude of momentum. Context will clarify.
\bar{p}	antiproton
π	symbol for pion (either π^0, π^+ , or π^-)
e^-	electron
e^+	positron
ν	neutrino
$\bar{\nu}$	antineutrino
ν_e	electron neutrino
$\bar{\nu}_e$	anti-electron neutrino
\mathbf{s}	intrinsic spin angular momentum
Q	charge, $Q = I_z + \frac{Y}{2}$
A	Baryon number
\mathbf{I}	isospin
I_z	z component of isospin
S	strange quantum number (strangeness)
C	charm quantum number (charmness)
B	bottom quantum number (bottomness)
T	top quantum number (topness)
Y	hypercharge, $Y = A + S + C + B + T = 2 < Q >$
\mathbf{s}	intrinsic spin angular momentum
P	parity
C_p	charge conjugation parity
G	G-parity

Abstract

The HZETRN code requires inclusive cross sections as input. One of the methods used to calculate these cross sections requires knowledge of all exclusive processes contributing to the inclusive reaction. Conservation laws are used to determine all possible exclusive reactions involving strong interactions between pions and nucleons. Inclusive particle masses are subsequently determined and are needed in cross-section calculations for inclusive pion production.

1. Introduction

For long duration space flight, it is important to be able to predict the radiation environment inside spacecraft. References 1 through 37 form a representative list of the literature of direct relevance to the discussion of exclusive cross sections for pions and nucleons pertinent to space radiation applications to space missions. One principal tool that has been used is the computer code HZETRN (refs. 1, 3, 4, and 5). In high energy interactions of cosmic rays, particles called mesons are copiously produced, but these particles have not yet been included in the HZETRN code. Work is currently underway to repair this deficiency, and the present work is part of this effort. The lightest meson is the pion, the most important of the mesons to be included, and is the subject of the present paper. The next heaviest meson is the kaon, and its interactions will be studied in future work.

An *exclusive* reaction is one in which all final state particles are specified, such as

$$A + B \rightarrow C + D \tag{1}$$

or

$$A + B \rightarrow C + D + E \tag{2}$$

or

$$A + B \rightarrow C + D + E + F \tag{3}$$

Let us suppose that only the above three exclusive reactions represent all of the possible processes leading to production of the particle C . All of these could be measured experimentally and calculated theoretically (i.e., from Feynman rules). The experiments would have to detect all the particles C, D, E, F , and a calculation would involve calculating each reaction separately for production of all particles C, D, E, F .

Experimentally, it may well be much easier to detect only the particle C of interest. In that case one measures the so-called *inclusive* reaction

$$A + B \rightarrow C + X \tag{4}$$

where X is *anything*. From the above analysis of the exclusive reactions, we know that X could be either D or $D+E$ or $D+E+F$. Even though it is easier experimentally to measure an inclusive cross section rather than an exclusive one, it is actually more difficult to

calculate with a theory based on Feynman rules because the theoretical calculation of the inclusive cross section is simply the sum of each of the separate exclusive reactions. Thus, the inclusive calculation involves much more work than a single exclusive calculation. To summarize, *inclusive cross sections are easier experimentally, whereas exclusive cross sections are easier theoretically.*

The HZETRN code requires only inclusive cross sections as input, and a major goal in space radiation research is to calculate such inclusive cross sections. A method for developing formulas for spectral and total inclusive cross sections has been developed previously (refs. 6–10). The method involves fitting curves to inclusive Lorentz invariant differential cross sections, which are then subsequently integrated to form inclusive spectral distributions and total cross sections. This method involves integrations over angle (to form spectral distributions) and over momentum (to form total cross sections), where the angle and momentum are for the particle C of interest. One integrates from zero to the maximum possible values of angle and momentum for particle C . *However, the formulas (ref. 6) for these maxima contain the mass of the particles X !* Thus, even though HZETRN requires only inclusive cross sections, it is nevertheless necessary to analyze the exclusive reactions contributing to the inclusive cross section. (This topic is discussed in more detail in Section 5 below.)

The goal of the present work is twofold. First, the aim is to use conservation laws to enumerate all possible exclusive processes contributing to the inclusive reaction. Second, the aim is to evaluate the masses of particles X based on the analysis of the exclusive reactions. These masses are then to be used in a separate work (ref. 6) to calculate inclusive cross sections.

This paper will concern itself only with pion interactions with nucleons. Furthermore, it will be limited only to strong interactions, as these are the most important from the point of view of space radiation shielding.

2. Interactions

According to table 1 the strong interaction cross section is about 10^6 times bigger than the weak interaction; thus, weak interaction *reactions* can be safely ignored for space radiation studies. The situation for electromagnetic (EM) interactions is a little more complicated. For the inelastic particle production processes to be considered, the EM cross section is actually tiny compared to the strong interaction because higher order tree level diagrams are involved. (Although for nuclear collisions, where the charge and therefore coupling constant is large, EM interactions need to be considered.) Thus, for meson production, only strong interaction reactions will be considered.

Table 1. Properties of the Fundamental Forces

Force	g	Decay type	τ	$c\tau$	Range	σ	Exchange particle
Strong	1	α	$\approx 10^{-23}$ sec	1 fm	1 fm	1 mb	pion/gluon
EM	10^{-2}	γ	$\approx 10^{-16}$ sec	100 Å	∞	∞	photon
Weak	10^{-6}	β	$\geq 10^{-10}$ sec	1 cm	10^{-3} fm	1 nb	W^\pm, Z^0
Gravity	10^{-43}				∞		graviton

References 2, 21, and 26 pertain to this table. Table 2 gives the conserved quantities and selection rules and tables 3 and 4 list the relevant quantum numbers.

g is the *approximate* coupling constant and gives relative strength.

τ is the typical lifetime.

σ is the typical interaction cross section, given roughly by the range squared.

Note: Weak interaction lifetimes actually vary all the way from 10^{-13} sec for the decay of the τ to 15 min for neutron decay (ref. 2).

However, for particle decays, the weak interaction is quite important. The quantity $c\tau$ gives the approximate distance a particle travels before decaying. This distance is tiny for strong and electromagnetic decays, and thus in these situations, one can consider the particle decaying instantaneously because it will not suffer another reaction before decaying. However, particles that decay weakly will travel a significant distance before decaying and therefore *can* interact again before decay. An example is pion decay. The π^0 decays electromagnetically and so can be considered to instantly disappear; however, the charged pions decay weakly and therefore the π^- will cause subsequent ionization. Thus, its stopping power needs to be included. The π^+ will almost certainly be annihilated with electrons before it has a chance to decay.

3. Inclusive Reactions

3.1. Nuclear Reactions

The HZETRN code requires spectral distributions $\frac{d\sigma}{dE}$ and total cross sections σ as input and are required only for *inclusive* reactions. The nuclear fragmentation reactions can be written

$$\begin{array}{lcl}
p + p & \rightarrow & p + X \\
& & n + X \\
n + p & \rightarrow & p + X \\
& & n + X \\
p + n & \rightarrow & p + X \\
& & n + X \\
n + n & \rightarrow & p + X \\
& & n + X \\
p + A & \rightarrow & p + X \\
& & n + X \\
& & A' + X \\
n + A & \rightarrow & p + X \\
& & n + X \\
& & A' + X \\
A + A & \rightarrow & p + X \\
& & n + X \\
& & A' + X
\end{array} \tag{5}$$

where p is a proton, n is a neutron, and A refers to any nucleus.

By writing the symbol N for a generic nucleon (either proton or neutron), the above can be written more compactly as

$$\begin{array}{lcl}
N + N & \rightarrow & N + X \\
N + A & \rightarrow & N + X \\
& & A' + X \\
A + A & \rightarrow & N + X \\
& & A' + X
\end{array} \tag{6}$$

The fragmentation reactions can be written even more compactly by using the symbol A for either a nucleus or a nucleon (single particle nucleus) as

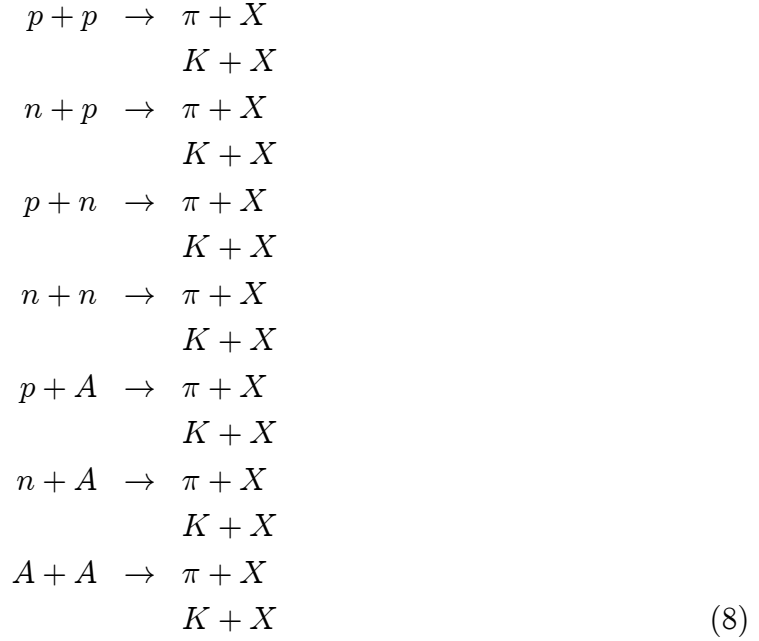
$$A + A \rightarrow A' + X \tag{7}$$

where A' also includes A . With this notation, a proton is a nucleus with $Z = 1$, written as ${}^A Z \equiv {}^1 1 \equiv {}^1 H \equiv p$, while a neutron is a nucleus with $Z = 0$, written as ${}^A Z \equiv {}^1 0 \equiv n$.

This condensed notation is useful for computer codes in which each individual reaction can be identified with an appropriate integer index.

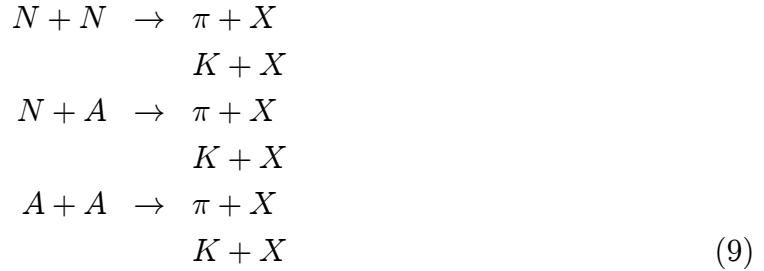
3.2. Particle Reactions

The meson production reactions are



where π is a pion and K is a kaon. The pion can be either π^0 , π^+ , or π^- . The kaon can be either K^0 , K^+ , or K^- . Note that the neutral kaon comes in two varieties, namely K_S^0 and K_L^0 . The neutral kaon contains a 50 percent admixture (ref. 15) of each of K_S^0 and K_L^0 .

The above can be written more compactly (with N representing both protons and neutrons) as



or even more compactly (with A representing nuclei, protons, and neutrons) as just

$$\begin{array}{l} A + A \rightarrow \pi + X \\ K + X \end{array} \quad (10)$$

A final simplification occurs if the mesons are written generically as M , representing either a pion or kaon. The above equation is then written

$$A + A \rightarrow M + X \quad (11)$$

Because HZETRN contains all possible particle interactions, one must also include the following reaction of the produced pions and kaons as well, namely

$$\begin{array}{l} \pi + p \rightarrow p + X \\ n + X \\ \pi + X \\ K + X \\ \pi + n \rightarrow p + X \\ n + X \\ \pi + X \\ K + X \\ \pi + A \rightarrow p + X \\ n + X \\ \pi + X \\ K + X \\ A' + X \end{array}$$

for the pions, and

$$\begin{array}{l} K + p \rightarrow p + X \\ n + X \\ \pi + X \\ K + X \\ K + n \rightarrow p + X \\ n + X \\ \pi + X \\ K + X \\ K + A \rightarrow p + X \end{array}$$

$$\begin{array}{l}
n + X \\
\pi + X \\
K + X \\
A' + X
\end{array}
\tag{12}$$

for the kaons.

Again, the above can be written more compactly (with N representing both protons and neutrons) as

$$\begin{array}{l}
\pi + N \rightarrow N + X \\
\pi + X \\
K + X \\
\pi + A \rightarrow N + X \\
\pi + X \\
K + X \\
A' + X
\end{array}$$

for the pions, and

$$\begin{array}{l}
K + N \rightarrow N + X \\
\pi + X \\
K + X \\
K + A \rightarrow N + X \\
\pi + X \\
K + X \\
A' + X
\end{array}
\tag{13}$$

for the kaons.

It can be written even more compactly (with A representing nuclei and protons and neutrons) as just

$$\begin{array}{l}
\pi + A \rightarrow \pi + X \\
K + X \\
A' + X
\end{array}$$

for the pions, and

$$\begin{array}{l}
K + A \rightarrow \pi + X \\
K + X \\
A' + X
\end{array}
\tag{14}$$

for the kaons.

A final simplification occurs if the mesons are written generically as M , representing either a pion or kaon. The preceding equation is then written

$$\begin{aligned} M + A &\rightarrow M + X \\ &A' + X \end{aligned} \tag{15}$$

3.3. Summary

Both the nuclear and particle reactions can be written together compactly as

$$A + A \rightarrow A' + X \tag{16}$$

$$M + X \tag{17}$$

and

$$M + A \rightarrow A' + X \tag{18}$$

$$M + X \tag{19}$$

where A is any nucleus, including a proton or neutron, and M is either a π^+ , π^- , π^0 , K^0 , K^+ , or K^- . Also $K^0 = 50$ percent $K_S^0 + 50$ percent K_L^0 . Note that A' can also be A .

The *nuclear* reactions (16) do not involve a meson M on either the left or right side of the reactions, whereas the *particle* reactions (17), (18), and (19) *do* involve M on *either* the left or right side of the reactions.

Such a compact way of writing the reactions is *useful* when keeping track of particles in a transport code such as HZETRN.

Note the vast multiplicity of all the above reactions because A can be any nucleus, and the pions and kaons can be charged or neutral. Other work (ref. 6) has shown that one needs to analyze exclusive reactions as well, which vastly increases the multiplicity. Examples of the lowest order reactions for nucleon-nucleon and pion-nucleon reactions are listed in table 5.

Finally, note that the *applied* space radiation transport problem differs from a *pure* particle physics investigation in that the reaction

$$\begin{aligned} M + M &\rightarrow M + X \\ &A + X \end{aligned} \tag{20}$$

might be of interest to particle physics but is not relevant to the space radiation problem.

4. Particle Decays

When pions or kaons are produced, they decay according to the following major decay modes. The charged pions undergo a *weak* decay

$$\pi^\pm \rightarrow \mu^\pm + \nu_\mu \quad (99.99 \text{ percent}) \quad \tau = 2.6 \times 10^{-8} \text{ sec}, \quad c\tau = 7.8 \text{ m} \quad (21)$$

where 99.99 percent refers to the percentage of this mode of decay relative to all possible decay modes. The lifetime τ has been listed as well as the quantity $c\tau$. Both quantities are in the rest frame of the decaying particle. The quantity $c\tau$ gives the approximate distance the pion will travel before decaying and is in the rest frame of the pion. Of course in the lab frame, which is the target frame or wall frame, the lifetime will appear longer and the distance will therefore be longer. Thus $c\tau$ is actually a minimum distance. Given that $c\tau \approx 8 \text{ m}$ for the above reaction, the charged pion will interact and produce ionization before decaying. For this reason, one must also include the stopping powers of charged pions in radiation transport codes.

The neutral pion undergoes an electromagnetic decay

$$\pi^0 \rightarrow 2\gamma \quad (99.8 \text{ percent}) \quad \tau = 8.4 \times 10^{-17} \text{ sec}, \quad c\tau = 25 \text{ nm} \quad (22)$$

Thus, the neutral pion can be considered not to propagate at all but to instantaneously produce its two photons γ . These photons will immediately decay via

$$\gamma \rightarrow e^+ + e^- \quad (23)$$

The electrons will produce Bremsstrahlung and also cause ionization. The positrons will annihilate

$$e^+ + e^- \rightarrow 2\gamma \quad (24)$$

to produce more photons which will decay producing more electrons and positrons. Thus a *cascade* of e^+e^- pairs and photons will be produced from the decaying neutral pion, which can be schematically written as

$$\pi^0 \rightarrow em \text{ cascade} \equiv e^+, e^-, \gamma \quad (25)$$

Let us now consider the kaon decays. The decay modes for the positive kaon are

$$K^\pm \rightarrow \mu^\pm + \nu_\mu \quad (64 \text{ percent}) \quad (26)$$

$$\pi^+ + \pi^0 \quad (21 \text{ percent}) \quad (27)$$

$$\pi^+ + \pi^+ + \pi^- \quad (6 \text{ percent}) \quad (28)$$

and the lifetime is

$$K^\pm \rightarrow \text{all decay modes} \quad \tau = 1.2 \times 10^{-8} \text{ sec}, \quad c\tau = 3.7 \text{ m} \quad (29)$$

The negative kaon decays in the same way except with the appropriate antiparticles. Given that the lifetime of the charged kaons is about the same as the charged pions, all our previous considerations for charged pion interactions also apply to the charged kaons.

Neutral kaons can be produced also in the reaction

$$p + p \rightarrow K^0 + X \quad (30)$$

That is how K^0 is *produced*, but it decays in a very odd fashion. The neutral kaon is an admixture of what are called K_{short} and K_{long} , denoted as K_S^0 and K_L^0 , respectively. The neutral kaon contains an equal admixture of these, namely

$$K^0 = 50 \text{ percent } K_S^0 + 50 \text{ percent } K_L^0 \quad (31)$$

These decay modes are as follows:

$$\begin{aligned} K_S^0 &\rightarrow \pi^+ + \pi^- && (69 \text{ percent}) \\ &2\pi^0 && (31 \text{ percent}) \end{aligned} \quad (32)$$

and the lifetime is

$$K_S^0 \rightarrow \text{all decay modes} \quad \tau = 0.9 \times 10^{-10} \text{ sec}, \quad c\tau = 2.7 \text{ cm} \quad (33)$$

and

$$\begin{aligned} K_L^0 &\rightarrow 3\pi^0 && (21 \text{ percent}) \\ &\pi^+ + \pi^- + \pi^0 && (13 \text{ percent}) \\ &\pi^\pm + \mu^\mp + \nu_\mu && (27 \text{ percent}) \\ &\pi^\pm + e^\mp + \nu_e && (39 \text{ percent}) \end{aligned} \quad (34)$$

with lifetime

$$K_L^0 \rightarrow \text{all decay modes} \quad \tau = 5.2 \times 10^{-8} \text{ sec}, \quad c\tau = 15.5 \text{ m} \quad (35)$$

Thus, the neutral kaon has a much longer lifetime than the neutral pion and will therefore need to be treated differently to the neutral pion in transport codes. In particular, propagation of the neutral kaon will have to be included. Furthermore, K_S^0 and K_L^0 will need to be treated differently.

The above considerations show why it is important to include pions and kaons in radiation transport codes.

5. Cross-Section Method

Solution of the Boltzmann equation (1) requires spectral distributions, $\frac{d\sigma}{dE}$ and total cross sections σ for pion and other particle production. In obtaining formulas for these cross sections, most previous work (refs. 7–10) has started with the Lorentz invariant differential cross section $\frac{d^3\sigma}{dp^3/E}$ which has then been integrated to form the spectral and total cross sections (ref. 6).

Consider the inclusive reaction

$$A + B \rightarrow C + X \quad (36)$$

where A and B represent the projectile and target, respectively, C is the produced particle of interest, and X is anything.

The spectral distribution for particle C is $\frac{d\sigma}{dE}$ and is related to the Lorentz invariant differential cross section by (refs. 6–8)

$$\frac{d\sigma}{dE_{lab}} = 2\pi p \int_0^{\theta_{max\ lab}} \frac{d^3\sigma}{dp^3/E} \sin \theta \, d\theta \quad (37)$$

where all variables refer to the produced particle C , and $\theta_{max\ lab}$ is the maximum angle of particle C in the lab frame given by (ref. 6)

$$\sin \theta_{max\ lab} = \frac{\sqrt{s} |\mathbf{p}_C^*|}{|\mathbf{p}_{lab}| m_C} \quad (38)$$

In carrying out integrations over angle, one uses $|\mathbf{p}_{max}^*|$ for $|\mathbf{p}_C^*|$, as discussed below.

The total cross section, calculated in the cm frame is (ref. 6)

$$\sigma = 2\pi \int_0^\pi d\theta \int_0^{p_{max}^*} dp \frac{d^3\sigma}{dp^3/E} \frac{p^2 \sin \theta}{\sqrt{p^2 + m^2}} \quad (39)$$

with p_{max}^* given by the square root of (ref. 6)

$$\mathbf{p}_C^{*2} = \frac{[s - (m_C + m_X)^2][s - (m_C - m_X)^2]}{4s} \quad (40)$$

The above expression in equation(40) for $|\mathbf{p}_C^*|$ is *not yet* the value to be substituted for p_{max}^* in equation (39). To obtain p_{max}^* one must explicitly determine the makeup of the particles X in reaction, in equation (36), and input the corresponding value of m_X

into equation (40). When this is done, *then* $|\mathbf{p}_C^*|$ will be the value for p_{max}^* to put into equations (38) and (39). Thus, one must determine the makeup of the X particles and therefore a study of the *exclusive* reactions leading to equation (36) must be undertaken, which is one of the main the purposes of the present work.

6. Exclusive Reactions

Simply having to determine the values of m_X leads to considerable work in determining all the different possible exclusive reactions. The primary tool for this study is analysis of all the conservation laws for particle reactions. The conserved quantities are listed in table 2, and the relevant quantum numbers are in tables 3 and 4. For *each* possible exclusive reaction, one must check that all of these laws are satisfied. Note that *all* quantities listed in table 2 are conserved in strong interactions, but may or may not be conserved in electromagnetic and weak interactions. In the present section, we deal with strong reactions in which all quantities are conserved. Later electromagnetic and weak decays will be considered.

6.1. Conservation Laws

In the present work, only strong hadron interactions are considered, and these will *include only pions and nucleons*. Thus, many of the conservation laws will simplify. Each conservation law will now be considered. Further details can be found in the appendix.

Energy E and linear momentum \mathbf{p} : For the reactions herein, these conservation laws determine the thresholds at which the various reactions occur. All thresholds are listed in table 5.

Because of the method used to parameterize cross sections (refs. 6–10), the threshold does not automatically appear in the parameterizations. When putting cross sections into a code like HZETRN, it is important to include a code statement that will set the cross section to zero when the energy is below threshold.

Angular momentum \mathbf{J} : One of the major constraints coming from angular momentum conservation is that fermions always occur in pairs (see appendix); however, this will not be relevant to strong reactions because fermions are not involved, but it will be relevant to weak decays. Other than that, \mathbf{J} is easy to conserve because particles can fly off in the appropriate angular momentum state \mathbf{L} to conserve \mathbf{J} (see appendix). Thus, in analyzing strong reactions, this conservation law need not be investigated in detail as particles will

automatically find the right L to ensure conservation. Thus, \mathbf{J} conservation need not be explicitly calculated.

Charge Q is the most important conservation law for strong reactions. Some of the other conservation laws below can be ignored if Q is conserved.

Lepton numbers L_e, L_μ, L_τ : This conservation law is not relevant to strong reactions because leptons are not involved, but it is very important in analyzing weak decays.

Baryon number A is a very important conservation law for strong reactions. It demands, for example, that *there must be an equal number of nucleons on both sides of a reaction*.

Isospin \mathbf{I} : Strong interactions conserve \mathbf{I} , but EM interactions do not.

z component of isospin I_z : As discussed in the appendix, conservation of \mathbf{I} implies conservation of I_z but not vice versa. Thus, because strong interactions conserve \mathbf{I} , they also conserve I_z automatically. In considering strong reactions, therefore, one only needs to explicitly consider conservation of \mathbf{I} . Conservation of I_z need not be calculated explicitly because it is guaranteed by \mathbf{I} conservation.

Strangeness S : In the present work, strange particles (e.g., kaons) are not considered in detail, so this conservation law can be ignored for the moment.

Flavor S, C, B, T : In the present work, flavored particles (e.g., kaons or charmed hadrons) are not considered in detail, and this conservation law can be ignored for the moment.

Hypercharge Y : The hypercharge is given by $Y = A + S + C + B + T$. The present work does not consider flavored particles in detail. Particles in the present work have $S = C = B = T = 0$, and thus $Y = A$. Conservation of baryon number will imply conservation of hypercharge. Thus, in the present work, conservation of Y need not be calculated explicitly because it is guaranteed by A conservation.

Parity P : As discussed in the appendix, parity conservation also must include the parity contribution from orbital angular momentum $(-1)^l$. Thus, as with conservation of \mathbf{J} above, one can safely assume the particles fly off in the right orbital angular momentum states to conserve both \mathbf{J} and P . Thus, P conservation need not be explicitly calculated.

Charge conjugation parity C : This is a multiplicative conservation law (as with all other parities as well). As seen in table 4, only the photon and π^0 are their own antiparticles and have well-defined charge conjugation parities. Photons won't occur in the strong reactions considered herein, and because the π^0 has $C_p = +1$, then charge conjugation parity conservation need not be explicitly calculated for strong reactions. Note, however, that this conservation law specifies why the π^0 *decays* (electromagnetically) into two photons and not three. For the reaction $\pi^0 \rightarrow \gamma + \gamma$ the charge conjugation parity of the

initial state is +1 and the charge conjugation parity of the final state is $-1 \times -1 = +1$.

CP need not be explicitly calculated.

Time reversal T need not be explicitly calculated.

CPT need not be explicitly calculated.

G parity is conserved whenever C and \mathbf{I} are conserved; thus, G parity conservation need not be explicitly calculated.

Summary: For the strong interactions between the (nonflavored) nucleons and pions considered in the present work, only conservation of charge Q , baryon number A , and isospin \mathbf{I} need to be explicitly calculated for each reaction.

6.2. Lowest Threshold Exclusive Reactions

The lowest threshold exclusive reactions are completely listed in table 5. It is a simple matter to check conservation of Q , A , and \mathbf{I} for each reaction. As mentioned above, conservation of baryon number simply means that there must be an equal number of nucleons in the initial and final states. Thus, conservation of charge and baryon number is easily checked by inspection of table 5. Let us then only consider a few examples of isospin conservation. For instance,

$$\begin{array}{l} p + p \rightarrow \pi^0 + p + p \\ \mathbf{I}: \quad \frac{1}{2} \quad \frac{1}{2} \rightarrow 1 \quad \frac{1}{2} \quad \frac{1}{2} \end{array}$$

$$\begin{array}{l} p + n \rightarrow \pi^+ + n + n \\ \mathbf{I}: \quad \frac{1}{2} \quad \frac{1}{2} \rightarrow 1 \quad \frac{1}{2} \quad \frac{1}{2} \end{array}$$

$$\begin{array}{l} \pi^0 + p \rightarrow \pi^- + p + \pi^+ \\ \mathbf{I}: \quad 1 \quad \frac{1}{2} \rightarrow 1 \quad \frac{1}{2} \quad 1 \end{array}$$

$$\begin{array}{l} \pi^- + n \rightarrow \pi^+ + n + \pi^- + \pi^- \\ \mathbf{I}: \quad 1 \quad \frac{1}{2} \rightarrow 1 \quad \frac{1}{2} \quad 1 \quad 1 \end{array}$$

where it is easily seen that \mathbf{I} is conserved in all previous reactions.

7. Results

The key results of this work are presented in table 5. These results include a complete list of all strong interactions between pions and nucleons. Table 5 shows the 48 different possible exclusive reactions within the constraints of the conservations laws. The other results presented in table 5 are the total mass of the particles produced in each exclusive reaction, represented as m_X . These reactions and the values of m_X are used in subsequent work (ref. 6) that calculates inclusive cross sections.

Table 2. Conserved Quantities and Selection Rules
[See refs. 17, 24, 25, 35, and 37]

+ denotes quantity is conserved, – denotes quantity is not conserved. (Quantities which are not conserved (–) does not mean that they are always violated. They may or may not be conserved in certain processes. However, quantities which are conserved (+), are always conserved.)

Conserved quantity	Strong	EM	Weak
Energy E	+	+	+
Linear momentum \mathbf{p}	+	+	+
Angular momentum \mathbf{J}	+	+	+
Charge Q	+	+	+
Electron lepton number L_e	+	+	+
Muon lepton number L_μ	+	+	+
Tau lepton number L_τ	+	+	+
Baryon number A	+	+	+
Isospin \mathbf{I}	+	-	-
z component isospin I_z	+	+	-
Strangeness S	+	+	-
Flavor (S, C, B, T)	+	+	-
Hypercharge Y	+	+	-
<u>Parities</u>			
Parity P	+	+	-
Charge conjugation C	+	+	-
CP or T	+	+	+ ¹
CPT	+	+	+
G parity	+	-	-

Recall (ref. 35): $Y \equiv A + S + C + B + T = 2 < Q >$ $Q = I_z + \frac{Y}{2}$
 $\eta_G = \eta_C(-1)^I$ (I = isospin of multiplet, $\eta_C \sim$ for neutral particle in multiplet.)

¹ CP or T is always conserved except for a small violation in K^0 weak decay.

Table 3. Particle Properties and Quantum Numbers

Particle	Quark content	Mass, MeV	s	Q	L_e	L_μ	L_τ	A	\mathbf{I}	I_z	S	C	B	T	Y
γ		0	1	0	0	0	0	0	0	0	0				
Z^0		91.1882 GeV	1	0	0	0	0	0	0	0	0				
W^+		80.419 GeV	1	0	0	0	0	0	0	0	0				
e^-		0.511	$\frac{1}{2}$	-1	+1	0	0	0			0				
e^+		0.511	$\frac{1}{2}$	+1	-1	0	0	0			0				
ν_e		≈ 0	$\frac{1}{2}$	0	+1	0	0	0			0				
$\bar{\nu}_e$		≈ 0	$\frac{1}{2}$	0	-1	0	0	0			0				
u			$\frac{1}{2}$	$+\frac{2}{3}$	0	0	0	$+\frac{1}{3}$	$\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0	$+\frac{1}{3}$
d			$\frac{1}{2}$	$-\frac{1}{3}$	0	0	0	$+\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	$+\frac{1}{3}$
s			$\frac{1}{2}$	$-\frac{1}{3}$	0	0	0	$+\frac{1}{3}$	0	0	+1	0	0	0	$+\frac{1}{3}$
c			$\frac{1}{2}$	$+\frac{2}{3}$	0	0	0	$+\frac{1}{3}$	0	0	0	+1	0	0	$+\frac{1}{3}$
b			$\frac{1}{2}$	$-\frac{1}{3}$	0	0	0	$+\frac{1}{3}$	0	0	0	0	+1	0	$+\frac{1}{3}$
t			$\frac{1}{2}$	$+\frac{2}{3}$	0	0	0	$+\frac{1}{3}$	0	0	0	0	0	+1	$+\frac{1}{3}$
π^0	$u\bar{u} + d\bar{d}$	134.98	0	0	0	0	0	0	1	0	0	0	0	0	0
π^+	$u\bar{d}$	139.57	0	+1	0	0	0	0	1	+1	0	0	0	0	0
π^-	$\bar{u}d$	139.57	0	-1	0	0	0	0	1	-1	0	0	0	0	0
p^+, N^+	uud	938.27	$\frac{1}{2}$	+1	0	0	0	+1	$\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0	+1
n, N^0	udd	939.57	$\frac{1}{2}$	0	0	0	0	+1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	+1
$\bar{p} = p^-$		938.27	$\frac{1}{2}$	-1	0	0	0	-1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0	-1
\bar{n}		939.57	$\frac{1}{2}$	0	0	0	0	-1	$\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0	-1
Δ^{++}	uuu	1232	0	+2	0	0	0	+1	$\frac{3}{2}$	$+\frac{3}{2}$	0	0	0	0	+1
Δ^+	uud	1232	0	+1	0	0	0	+1	$\frac{3}{2}$	$+\frac{1}{2}$	0	0	0	0	+1
Δ^0	udd	1232	0	0	0	0	0	+1	$\frac{3}{2}$	$-\frac{1}{2}$	0	0	0	0	+1
Δ^-	ddd	1232	0	-1	0	0	0	+1	$\frac{3}{2}$	$-\frac{3}{2}$	0	0	0	0	+1

*References for this table are 13, 15, and 16.

Table 4. Particle Parities and Antiparticles and Forces Experienced
(s = Strong, w = weak, em = electromagnetic)

Particle	Antiparticle	P	C_p	$C_p P$	G	
γ	γ (SELF)	-1	-1	+1		em
Z^0	Z^0 (SELF)					w
W^+	W^-					em, w
e^-	e^+	+1				em, w
e^+	e^-	-1				em, w
ν_e	$\bar{\nu}_e$	+1				w
$\bar{\nu}_e$	ν_e					w
u	\bar{u}	+1				s, em, w
d	\bar{d}	+1				s, em, w
s	\bar{s}	+1				s, em, w
c	\bar{c}	+1				s, em, w
b	\bar{b}	+1				s, em, w
t	\bar{t}	+1				s, em, w
π^0	π^0 (SELF)	-1	+1	-1	-1	s, em, w
π^+	π^-	-1			-1	s, em, w
π^-	π^+	-1			-1	s, em, w
p^+, N^+	p^-	+1				s, em, w
n, N^0	\bar{n}	+1				s, em, w
$\bar{p} = p^-$	p^+	+1				s, em, w
\bar{n}	n	+1				s, em, w
Δ^{++}		+1				s, em, w ?
Δ^+		+1				s, em, w ?
Δ^0		+1				s, em, w?
Δ^-		+1				s, em, w ?

Note: The time reversal operator T is antiunitary and therefore does not have a quantum number associated with it (ref. 12). Thus, the time parity quantum number T_p does not exist; therefore, a corresponding $C_p P T_p$ quantum number does not exist either. Nevertheless, it is still possible to analyze T and CPT invariance in particle reactions.

Note: A charge conjugation parity quantum number cannot be associated with charged particles (ref. 25) because, for example, $C|\pi^+ \rangle \equiv |\pi^- \rangle$ and thus $C|\pi^+ \rangle \neq \pm |\pi^+ \rangle$

References for this table are 15, 16, 22, 26, and 35.

In table 5 $KE_{1\,lab}$ is the kinetic energy threshold, $p_{1\,lab}$ is the momentum threshold, and \sqrt{s} is the total cm energy at threshold. All energies and masses are in MeV . If no numbers are listed it means that the sum of the final state masses is smaller than the initial state masses, implying no threshold. The mass of the deuteron d used is 1876.125 MeV.

Table 5. The m_X for Minimum Threshold Particle Reactions[†] Occurring via the Strong Interaction

Reaction $A + B \rightarrow C + X$	$KE_{1\,lab}$	$p_{1\,lab}$	\sqrt{s}	m_X
$p + p \rightarrow p + p$	0	0	1876.5	938.27
$n + p + \pi^+$	292.31	796.23	2017.4	1077.8
$\pi^0 + p + p$	279.67	776.55	2011.5	1876.5
$\pi^+ + p + n$	292.31	796.23	2017.4	1877.8
$\pi^+ + d$	288.63	790.53	2015.7	1876.1
$\pi^- + p + p + \pi^+$	599.8	1218.7	2155.7	2016.1
$n + n \rightarrow p + n + \pi^-$	286.71	788.02	2017.4	1079.1
$n + n$	0	0	1879.1	939.57
$\pi^0 + n + n$	279.66	776.99	2014.1	1879.1
$\pi^+ + \pi^- + n + n$	599.75	1219.3	2158.3	2018.7
$\pi^- + p + n$	286.71	788.02	2017.4	1877.8
$\pi^- + d$	283.03	782.28	2015.7	1876.1
$p + n \rightarrow p + n$	0	0	1877.8	939.57
$n + p$	0	0	1877.8	938.27
$\pi^0 + p + n$	279.47	776.23	2012.8	1877.8
$\pi^0 + d$	275.8	770.46	2011.1	1876.1
$\pi^+ + n + n$	292.11	795.91	2018.7	1879.1
$\pi^- + p + p$	286.52	787.25	2016.1	1876.5

[†]Particle C is the produced particle of interest and is the first particle listed on the right-hand side of each reaction. Particles X are all the remaining particles; m_X is the sum of the masses of these remaining particles.

Table 5. (continued)

$\pi^0 + p \rightarrow$	$p + \pi^0$	0	0	1073.3	134.98
	$n + \pi^+$	6.7558	43.237	1079.1	139.57
	$\pi^0 + p$	0	0	1073.3	938.27
	$\pi^+ + n$	6.7558	43.237	1079.1	939.57
	$\pi^- + p + \pi^+$	175.97	280.13	1217.4	1077.8
$\pi^0 + n \rightarrow$	$p + \pi^-$	3.7684	32.117	1077.8	139.57
	$n + \pi^0$	0	0	1074.6	134.98
	$\pi^0 + n$	0	0	1074.6	939.57
	$\pi^+ + n + \pi^-$	175.93	280.08	1218.7	1079.1
	$\pi^- + \pi^+ + n$	175.93	280.08	1218.7	1079.1
$\pi^+ + p \rightarrow$	$p + \pi^+$	0	0	1077.8	139.57
	$n + \pi^+ + \pi^+$	172.4	279.01	1218.7	279.14
	$\pi^0 + p + \pi^+$	164.77	270.45	1212.8	1077.8
	$\pi^+ + p$	0	0	1077.8	938.27
	$\pi^- + p + \pi^+ + \pi^+$	362.19	481.95	1357.0	1217.4
$\pi^+ + n \rightarrow$	$p + \pi^0$	-	-	-	134.98
	$n + \pi^+$	0	0	1079.1	139.57
	$\pi^0 + p$	-	-	-	938.27
	$\pi^+ + n$	0	0	1079.1	939.57
	$\pi^- + p + \pi^+$	168.98	275.18	1217.4	1077.8
$\pi^- + p \rightarrow$	$p + \pi^-$	0	0	1077.8	139.57
	$n + \pi^0$	-	-	-	134.98
	$\pi^0 + n$	-	-	-	939.57
	$\pi^+ + n + \pi^-$	172.4	279.01	1218.7	1079.1
	$\pi^- + p$	0	0	1077.8	938.27
$\pi^- + n \rightarrow$	$p + \pi^- + \pi^-$	168.98	275.18	1217.4	279.14
	$n + \pi^-$	0	0	1079.1	139.57
	$\pi^0 + n + \pi^-$	164.73	270.4	1214.1	1079.1
	$\pi^+ + n + \pi^- + \pi^-$	362.07	481.83	1358.3	1218.7
	$\pi^- + n$	0	0	1079.1	939.57

8. Appendix: Review of Conservation Laws

Reference 12 (Frauenfelder and Henley) has three chapters devoted to conservation laws and is an excellent reference. Another excellent reference is Griffiths (ref. 2).

8.1. Crossing Symmetry

A good reference for this section is Griffiths (ref. 2). Suppose the following reaction occurs:

$$A + B \rightarrow C + D \quad (41)$$

Now, from the conservation laws to be discussed, the *reverse reaction* can also occur:

$$C + D \rightarrow A + B \quad (42)$$

However, the principle of *crossing symmetry* says that the following reactions will also occur, *provided that energy and momentum are still conserved*:

$$\begin{aligned} A &\rightarrow \bar{B} + C + D \\ A + \bar{C} &\rightarrow \bar{B} + D \\ \bar{C} + \bar{D} &\rightarrow \bar{A} + \bar{B} \end{aligned} \quad (43)$$

where an overhead bar denotes the antiparticle. Thus, when a particle is “crossed” over to the other side, it is changed to an antiparticle.

As Griffiths (ref. 2) points out, the principle of crossing symmetry tells us that Compton scattering

$$\gamma + e^- \rightarrow \gamma + e^- \quad (44)$$

is “really” the same process as pair annihilation,

$$e^- + e^+ \rightarrow \gamma + \gamma \quad (45)$$

Thus, we can see how powerful this principle of crossing symmetry really is.

8.2. Conservation of Lepton Number

It is actually the antineutrino that is involved in beta decay of the neutron

$$n \rightarrow p^+ + e^- + \bar{\nu} \quad (46)$$

Given that the neutrino is neutral, it is natural to ask whether the neutrino is its own antiparticle. (After all, the π^0 and the photon are their own antiparticles; however, the

neutron is not.) As Griffiths (ref. 2) states, “from the positive results of Cowan and Reines, we know that the crossed reaction

$$\nu + n \rightarrow p^+ + e^- \quad (47)$$

must also occur at about the same rate. Davis looked for the analogous reaction using antineutrinos:

$$\bar{\nu} + n \rightarrow p^+ + e^- \quad (48)$$

He found that this reaction does *not* occur and thus established that the neutrino and antineutrino are distinct particles” (ref. 2).

How does one account for this particle? Introduce a new quantum number (call it lepton number L) and a *corresponding* new conservation law (conservation of lepton number). Neutrinos are extremely light particles and therefore deserve to be called leptons. Assign lepton number $L = +1$ to leptons and $L = -1$ to antileptons. Consider the previous reactions with this new conservation law,

$$\begin{aligned} n &\rightarrow p^+ + e^- \\ L : 0 &\neq 0 + 1 \end{aligned} \quad (49)$$

which violates conservation of lepton number and therefore the reaction will not occur,

$$\begin{aligned} n &\rightarrow p^+ + e^- + \bar{\nu} \\ L : 0 &= 0 + 1 - 1 \end{aligned} \quad (50)$$

which obeys conservation of lepton number and therefore the reaction will occur,

$$\begin{aligned} \bar{\nu} + p^+ &\rightarrow n + e^+ \\ L : -1 + 0 &= 0 - 1 \end{aligned} \quad (51)$$

which obeys conservation of lepton number and therefore the reaction will occur,

$$\begin{aligned} \nu + n &\rightarrow p^+ + e^- \\ L : +1 + 0 &= 0 + 1 \end{aligned} \quad (52)$$

which obeys conservation of lepton number and therefore the reaction will occur,

$$\begin{aligned} \bar{\nu} + n &\rightarrow p^+ + e^- \\ L : -1 + 0 &\neq 0 + 1 \end{aligned} \quad (53)$$

which violates conservation of lepton number and therefore the reaction will not occur.

8.2.1. Muon Lepton Number and Tau Lepton Number

Thus, all seems fine with electrons and neutrinos, but what about muons and taus? It turns out that the reaction

$$\mu^- \rightarrow e^- + \gamma \quad (54)$$

is never observed experimentally, even though it does not violate conservation of lepton number. To account for muons and taus three different types of lepton numbers are introduced, namely conservation of electron lepton number L_e , conservation of muon lepton number L_μ , and conservation of tau lepton number L_τ . The lepton number that we have been studying should be called *electron lepton number*, L_e . Thus, the reaction

$$\begin{aligned} \mu^- &\rightarrow e^- + \gamma \\ L_e : 0 &\neq +1 + 0 \\ L_\mu : +1 &\neq 0 + 0 \end{aligned} \quad (55)$$

violates conservation of electron lepton number and muon lepton number. Then there is the question of how to explain the observed decay (ref. 2)

$$\mu^- \rightarrow e^- + \bar{\nu} + \nu \quad (56)$$

The answer is that there must be different *types* of neutrinos, each associated with the electron, muon, or tau. In the days before the tau was known, this decay was referred to as the *two-neutrino hypothesis*. Today we call the three types of neutrinos the electron neutrino ν_e , the muon neutrino ν_μ , and the tau neutrino ν_τ , each with its own different lepton number. Thus, the above reaction is actually

$$\begin{aligned} \mu^- &\rightarrow e^- + \bar{\nu}_e + \nu_\mu \\ L_e : 0 &= 1 - 1 + 0 \\ L_\mu : +1 &= 0 + 0 + 1 \end{aligned} \quad (57)$$

β decay is more correctly written as

$$\begin{aligned} n &\rightarrow p + e^- + \bar{\nu}_e \\ L_e : 0 &= 0 + 1 - 1 \\ L_\mu : 0 &= 0 + 0 + 0 \\ L_\tau : 0 &= 0 + 0 + 0 \end{aligned} \quad (58)$$

8.3. Conservation of Baryon Number

In 1938 Stuckelberg introduced the law of conservation of baryon number to account for the stability of the proton. The reaction

$$p^+ \rightarrow e^+ + \gamma \tag{59}$$

is allowed by all the conservation laws above, yet does not occur in nature. (As Griffiths (ref. 2) points out, this reaction does violate conservation of lepton number, but that violation was not known until later. To properly include lepton number conservation, just put in a neutrino on the right-hand side.) Stuckelberg proposed a new conservation law and consequently the existence of a new quantum number, which is now called baryon number. All baryons are assigned a baryon number $A = +1$; all antibaryons are assigned $A = -1$; and all mesons and leptons are assigned $A = 0$. Therefore the proton is stable by virtue of the conservation of baryon number.

8.4. Strange Particles

Muirhead (ref. 13) contains some excellent examples relevant to this section. As Griffiths states, “for a brief period in 1947, it was possible to believe that the major problems of elementary particle physics were solved” (ref. 2). Yukawa’s meson and Dirac’s positron had been found, and people pretty much believed in Pauli’s neutrino hypothesis, only awaiting experimental confirmation. No one really understood why the muon existed, but let’s ignore that.

Late in 1947, yet another dramatic discovery was made in the cosmic rays. The so-called V particles were discovered (now called kaons K , lambdas Λ , cascades Ξ , and sigmas Σ), which, due to their pion decays, left tracks in the shape of a V in nuclear emulsions. When these V particles were able to be produced in accelerators they were also called strange particles because of the following (ref. 28):

- They were always produced in pairs.
- They were all produced from reactions where the initial state contained protons or neutrons, but in some decays there would be no protons or neutrons in the final state.
- All the strange particles are unstable.
- All strange particles are produced via strong interactions but often decay via weak interactions.

Because they are produced copiously but decay only slowly suggested that their production mechanism is different from their decay mechanism. A copious production rate indicates production via the strong force, but a slow decay rate suggests decay via the weak force. (They decayed even more slowly than typical EM lifetimes.) To account

for all this, Gell-Mann introduced a new quantum number which he called strangeness S (don't confuse this with spin s), which he guessed is conserved in strong interactions but not in weak interactions. The strange particles are strongly produced and are always produced in pairs such as

$$\begin{aligned}\pi^- + p^+ &\rightarrow K^0 + \Lambda \\ S : 0 + 0 &= +1 - 1\end{aligned}\tag{60}$$

where the kaons carry $S = +1$ and the lambdas carry $S = -1$. Now that the strange particles have been produced, they will decay. But the kaon is the lightest strange meson, and the lambda is the lightest strange baryon, so they can't decay into other strange particles. They can only decay into nonstrange particles. Now they can't decay via the strong interaction because it conserves strangeness. If strangeness is *always* conserved by every interaction, then they won't decay at all. But they *are* observed to decay, and thus it must be via the EM or weak force. The decay rate is too long for the EM force and so their decay must be via the weak force, and because they decay means that the weak force does not conserve strangeness. The lambda decay is

$$\begin{aligned}\Lambda &\rightarrow p^+ + \pi^- \\ S : -1 &\neq 0 + 0\end{aligned}\tag{61}$$

8.5. Other Conservation Laws

We have now introduced the complete family of leptons and some of the hadrons. We have discussed the three forces, namely electromagnetic, strong, and weak. (Gravitational interactions between elementary particles are so incredibly weak as to be entirely negligible at energies accessible to accelerators today.) Now we turn to the question of what reactions and decays are possible between the particles. We will only consider whether reactions or decays are possible or not. Feynman diagram techniques can be used to calculate the cross sections and decay rates (lifetimes) of these processes.

8.5.1. Conservation of 4-Momentum for Decays

Conservation of 4-momentum includes conservation of energy and conservation of linear 3-momentum. They are best dealt with as conservation of the total 4-momentum. This conservation law is very important for reactions as it determines the threshold energy below which a certain reaction cannot occur. These constraints will be calculated below. Conservation of energy and momentum is also important for particle decays. An important result is that the *masses of the decay products cannot exceed the mass of the decaying particle*, which is now proved. Consider the decay

$$1 \rightarrow 2 + 3 + 4 + 5 + \cdots\tag{62}$$

Conservation of 4-momentum yields

$$p_1^2 = (p_2 + p_3 + p_4 + p_5 + \cdots)^2 \quad (63)$$

In the cm frame of the decaying particle, this gives

$$m_1 = E_2 + E_3 + E_4 + E_5 + \cdots \quad (64)$$

However, in general,

$$E_i \geq m_i$$

and thus $E_2 + E_3 + E_4 + E_5 + \cdots \geq m_2 + m_3 + m_4 + m_5 + \cdots$ gives

$$m_1 \geq m_2 + m_3 + m_4 + m_5 + \cdots \quad (65)$$

showing that the masses of the decay products cannot exceed the mass of the decaying particle.

8.4.2. Conservation of 4-Momentum for 3-Body Reactions

For the 3-body decay,

$$1 \rightarrow 2 + 3 \quad (66)$$

The previous result gives

$$m_1 \geq m_2 + m_3 \quad (67)$$

For a 3-body *reaction*

$$2 + 3 \rightarrow 1 \quad (68)$$

The result (eq. (67)) obviously remains true. Thus, for a 3-body reaction, *the mass of the final state must be greater than the masses of the two initial particles.*

For this reason, a reaction such as

$$\pi^0 + p \rightarrow p \quad (69)$$

is forbidden due to conservation of 4-momentum. However, the reaction

$$\pi^0 + p \rightarrow \Delta^+ \quad (70)$$

is allowed.

8.5.3. Conservation of 4-Momentum and Threshold Energy

Because of the method used to parameterize cross sections (refs. 6–10), the threshold does not appear automatically in the parameterizations. When putting cross sections into a code like HZETRN, it is important to include a code statement that will set the cross section to zero when the energy is below threshold.

The reaction threshold energy is now derived. Consider the reaction

$$1 + 2 \rightarrow 3 + 4 + 5 + 6 + \cdots \quad (71)$$

The Mandelstam invariant s is

$$\begin{aligned} s &\equiv (p_1^\mu + p_2^\mu)^2 = (p_3^\mu + p_4^\mu + p_5^\mu + p_6^\mu + \cdots)^2 \\ &= (E_1 + E_2)^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 \\ &= (E_3 + E_4 + E_5 + E_6 + \cdots)^2 - (\mathbf{p}_3 + \mathbf{p}_4 + \mathbf{p}_5 + \mathbf{p}_6 + \cdots)^2 \end{aligned} \quad (72)$$

The *particle production threshold* is defined so that all the final state particles are at rest,

$$\mathbf{p}_3 = \mathbf{p}_4 = \mathbf{p}_5 = \mathbf{p}_6 = \cdots = 0 \quad (73)$$

Which implies the *cm* frame because $\mathbf{p}_3 + \mathbf{p}_4 + \mathbf{p}_5 + \mathbf{p}_6 + \cdots = 0 + 0 + 0 + 0 + \cdots = 0$. Thus, at threshold, the Mandelstam invariant s is

$$s = (m_3 + m_4 + m_5 + m_6 + \cdots)^2 \quad (74)$$

Because s is a relativistic invariant, we can equate it to its *lab* value so that

$$E_{1 \text{ lab threshold}} = \frac{(m_3 + m_4 + m_5 + m_6 + \cdots)^2 - m_1^2 - m_2^2}{2m_2} \quad (75)$$

giving the threshold kinetic energy as

$$\begin{aligned} T_{1 \text{ lab threshold}} &= \frac{(m_3 + m_4 + m_5 + m_6 + \cdots)^2 - m_1^2 - m_2^2}{2m_2} - m_1 \\ &= \frac{(m_3 + m_4 + m_5 + m_6 + \cdots)^2 - m_1^2 - m_2^2 - 2m_1m_2}{2m_2} \\ &= \frac{(m_3 + m_4 + m_5 + m_6 + \cdots)^2 - (m_1 + m_2)^2}{2m_2} \end{aligned} \quad (76)$$

or

$$T_{1 \text{ lab threshold}} = \frac{(\sum_f m_f)^2 - (m_1 + m_2)^2}{2m_2} \quad (77)$$

where $\sum_f m_f$ denotes the sum over the masses of all final state particles.

8.5.4. Conservation of Charge

Charge is always conserved in elementary particle reactions and decays; thus, in β decay

$$\begin{aligned} n &\rightarrow p^+ + e^- + \bar{\nu} \\ Q : 0 &= +1 - 1 + 0 \end{aligned} \quad (78)$$

8.5.5. Conservation of Angular Momentum

The best reference for this section is Muirhead (ref. 13). Another good reference is Roe (ref. 14). By knowing that the photon is a spin-1 particle, the angular momentum conservation laws, $\Delta m = \pm 1, 0$ and $\Delta L = \pm 1$, for dipole transitions can be derived for atomic, molecular, or nuclear photon transitions.

In elementary particle reactions, the incoming or outgoing particles can be a variety of \mathbf{L} states, and thus angular momentum does not constrain things very much. Remember it is $\mathbf{J} = \mathbf{L} + \mathbf{s}$ which is to be conserved, which is always easy because \mathbf{L} can be anything (i.e., particles can fly off in S or P or D states, and so on).

However, there *is* one important constraint from angular momentum and that is

Fermions always occur in pairs.

The reason (ref. 13) for this pairing is that orbital angular momentum is always an integer whereas the intrinsic spin angular momentum of fermions is always half integer. The total angular momentum will only be conserved if fermions occur in pairs. Examples are the reactions

$$\begin{aligned} n &\rightarrow p^+ + e^- + \bar{\nu} \\ \mathbf{s} : \frac{1}{2} &\rightarrow \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \end{aligned} \quad (79)$$

Of course one does *not* read $\frac{3}{2}$ on the right. Because we are considering vectors, the three spin $\frac{1}{2}$ values on the right-hand side can orient to equal the spin $\frac{1}{2}$ on the left. Also

$$\begin{aligned} p + p &\rightarrow p + p + \pi^0 \\ \mathbf{s} : \frac{1}{2} \quad \frac{1}{2} &\rightarrow \frac{1}{2} \quad \frac{1}{2} \quad 0 \end{aligned} \quad (80)$$

where the pion (π^0) has an intrinsic spin of 0. In fact, a spin is *measured* in this way. By measuring the angular distribution of pions (suppose they come off spherically symmetric, which corresponds to an S state), then one can deduce that pions have spin 0.

Another example is seen in reference 13:

$$\begin{aligned} \pi^- + p &\rightarrow n + \gamma \\ \mathbf{s} : 0 \quad \frac{1}{2} &\rightarrow \frac{1}{2} \quad 1 \end{aligned} \quad (81)$$

where the photon has spin 1.

8.5.6. Conservation of Isospin

Isospin is an important conservation law and can be summarized as

I is conserved in strong interactions (not EM or weak).

I_z is conserved in strong and EM interactions (not weak).

Both Beiser (ref. 33) and Muirhead (ref. 13) have excellent discussions of isospin. The concept of isospin was introduced by Heisenberg to account for the near equality of the masses of the proton and neutron and the charge independence of the nuclear force. That is, the strength of the nn, pp, and np forces are observed to be identical if one disregards electromagnetic effects. Isospin is a quantum number analogous to ordinary intrinsic spin. A fermion has intrinsic spin $s = \frac{1}{2}$, and the z -component can be $s = +\frac{1}{2}$ or $s = -\frac{1}{2}$, which we often describe as spin “up” or spin “down.” Heisenberg pictured the proton and neutron as belonging to a single isospin $\mathbf{I} = \frac{1}{2}$ doublet, but the isospin “up” state is the proton, and the isospin “down” state is the neutron. Thus the proton has $I_z = +\frac{1}{2}$, while the neutron has $I_z = -\frac{1}{2}$. Thus, the isospin assignment serves to distinguish the charges of the neutron and proton. The charges are given by

$$Q = I_z + \frac{1}{2} \quad (82)$$

with Q given in units of e . This equation (82) is generalized to what is called the Gell-Mann–Nishijima relation

$$Q = I_z + \frac{A}{2} + \frac{S}{2} \quad (83)$$

where A is the baryon number (+1 for both proton and neutron) and S is the strangeness (0 for proton and neutron). Given that charge Q is always conserved and baryon number A is always conserved, then I_z *will be conserved only when S is conserved*. Thus, because weak interactions violate strangeness conservation they will also violate I_z conservation.

Also, the Gell-Mann–Nishijima relation will later be generalized to

$$Q = I_z + \frac{Y}{2} \quad (84)$$

where Y is called the strong hypercharge. (For the above cases $Y = A + S$, but this equation is further generalized to $Y = A + S + C + B + T$).

Isospin is also a useful quantity for the three pions (π^+ , π^- , π^0), which also have very similar masses. The way to distinguish them is obviously to assign the pion multiplet an

isospin $\mathbf{I} = 1$ with each separate pion having a different value of I_z . The assignment is $\pi^+(I_z = +1), \pi^-(I_z = -1), \pi^0(I_z = 0)$. The pions have $B = S = 0$, and therefore

$$Q = I_z \quad (85)$$

for pions. (Again, Q is in units of e .)

Both I and I_z are conserved in strong interactions (which is the whole point of introducing them. Neither is always conserved in weak interactions. I is not conserved in EM interactions, but I_z is conserved in EM interactions. Let us look at some examples (ref. 33). The first is a *strong* interaction process

$$\begin{array}{rcl} p^+ + p^+ & \rightarrow & \Lambda^0 + K^0 + p^+ + \pi^+ \\ \mathbf{I}: \quad \frac{1}{2} \quad \frac{1}{2} & \rightarrow & 0 \quad \frac{1}{2} \quad \frac{1}{2} \quad 1 \\ I_z: \quad \frac{1}{2} \quad + \frac{1}{2} & = & 0 \quad -\frac{1}{2} \quad + \frac{1}{2} \quad + 1 \end{array} \quad (86)$$

in which both \mathbf{I} and I_z are conserved. Note that the quantum number I_z is simply additive. As with spin \mathbf{s} , the conservation of \mathbf{I} involves vector addition, so the numbers do not simply add. Instead they add as vectors (ref. 33).

Now consider an EM process, which is the famous decay of the neutral pion into two photons

$$\begin{array}{rcl} \pi^0 & \rightarrow & \gamma + \gamma \\ \mathbf{I}: \quad 1 & \rightarrow & 0 \quad 0 \\ I_z: \quad 0 & = & 0 \quad + 0 \end{array} \quad (87)$$

We see that \mathbf{I} is not conserved, but I_z is conserved.

Finally, consider the following two *weak* decay processes

$$\begin{array}{rcl} \Lambda^0 & \rightarrow & n + \pi^0 \\ \mathbf{I}: \quad 0 & \rightarrow & \frac{1}{2} \quad 1 \\ I_z: \quad 0 & \neq & -\frac{1}{2} \quad + 0 \end{array} \quad (88)$$

and

$$\begin{array}{rcl} \Lambda^0 & \rightarrow & p^+ + \pi^- \\ \mathbf{I}: \quad 0 & \rightarrow & \frac{1}{2} \quad 1 \\ I_z: \quad 0 & \neq & \frac{1}{2} \quad - 1 \end{array} \quad (89)$$

where neither \mathbf{I} nor I_z is conserved.

Finally, note that

If \mathbf{I} is conserved then I_z is conserved, but not vice versa.

The reason for this relationship is simply from vector addition. The vector equation

$$\mathbf{A} + \mathbf{B} = \mathbf{C} + \mathbf{D} \quad (90)$$

implies equality of all components, namely

$$A_x + B_x = C_x + D_x \quad (91)$$

$$A_y + B_y = C_y + D_y \quad (92)$$

$$A_z + B_z = C_z + D_z \quad (93)$$

Isospin conservation can be written

$$\sum_i \mathbf{I}_i = \sum_f \mathbf{I}_f \quad (94)$$

with a sum over all initial state and final state particles involved in the reaction. This vector addition formula then implies conservation of the z component,

$$\sum_i I_{zi} = \sum_f I_{zf} \quad (95)$$

However, of course, this latter equation does *not* imply the former, as seen with electromagnetic interactions which conserve I_z but not \mathbf{I} . Strong interactions, on the other hand, conserve \mathbf{I} and therefore must also conserve I_z .

Stating this another way, equation (90) does imply equations (91), (92), and (93), but equation (93) does *not* imply (91) and (92); that is, equation (93) does not imply equation (90).

A more rigorous approach to this statement is based on commutation with the Hamiltonian. All conserved quantities commute with the Hamiltonian. Thus, conservation of \mathbf{I} is expressed as

$$[H, \mathbf{I}] = 0 \quad (96)$$

which of course implies

$$[H, I_x]\mathbf{i} + [H, I_y]\mathbf{j} + [H, I_z]\mathbf{k} = 0 \quad (97)$$

which is only satisfied if

$$[H, I_x] = 0 \quad (98)$$

$$[H, I_y] = 0 \quad (99)$$

$$[H, I_z] = 0 \quad (100)$$

implying conservation of I_z . As before, equation (96) does imply equations (98), (99), and (100), but equation (100) does *not* imply (98) and (99); that is, equation (100) does not imply equation (96).

8.5.7. Hypercharge

Another quantum number that is often used is called hypercharge. *Hypercharge is often used instead of Strangeness*. Actually, its correct name is *strong hypercharge*, as opposed to weak hypercharge. Strong hypercharge is defined as (ref. 12)

$$Y \equiv A + S + C + B + T = 2 < Q > \quad (101)$$

with Q in units of e and where A is baryon number, S is strangeness, C is charmness, B is bottomness, and T is topness. All the particles we have discussed so far have zero values for charm, bottomness, and topness. The quantity $< Q >$ represents the *average charge of a multiplet*. Thus, the average charge of the pion multiplet is 0 and therefore $Y = 0$. The average charge of the nucleon multiplet is +1, and so $Y = \frac{1}{2}$ for the nucleon (for both proton and neutron).

Often books (ref. 33) write it simply as $Y \equiv A + S$ because most of the particles have $C = B = T = 0$, but it is better to just use the general formula given previously so that all cases are covered. Thus, the Gell-Mann–Nishijima relation should be generalized to (refs. 12 and 23)

$$Q = I_z + \frac{Y}{2} \quad (102)$$

The *Flavor* quantum number represents S, C, B, T . *Flavor is conserved in strong and EM interactions but not in weak interactions* (ref. 2). Given that baryon number A is always conserved, it follows that Y is conserved the same way as for flavor. Thus, Y is *conserved in strong and EM interactions but not in weak interactions*.

8.5.8. Conservation of Parity

Some excellent references for this section are these: 2, 12, 23, 24, and 25. Parity is the operation which inverts spatial coordinates; that is, $(x, y, z) \rightarrow (-x, -y, -z)$ or $\mathbf{r} \rightarrow -\mathbf{r}$.

It is an example of a discrete as opposed to continuous transformation. In particular, the parity operator P acting on a wave function (actually *any* function) is

$$P\psi(\mathbf{r}) \equiv \psi(-\mathbf{r}) \quad (103)$$

Define an eigenvalue equation

$$P\psi(\mathbf{r}) \equiv \eta\psi(\mathbf{r}) \quad (104)$$

where η is the *eigenvalue or parity quantum number*. Now let P act again as in

$$\begin{aligned} P^2\psi(\mathbf{r}) &= P\psi(-\mathbf{r}) = \psi(\mathbf{r}) \\ &= \eta P\psi(\mathbf{r}) = \eta^2\psi(\mathbf{r}) \end{aligned} \quad (105)$$

Thus $\eta^2 = 1$ or

$$\eta = \pm 1 \quad (106)$$

(Because $P^2 = 1$, we say that P is idempotent.)

Now the wave function is written generally as (refs. 24 and 25)

$$\begin{aligned} \psi(r, \theta, \phi) &= R_{nl}(r)Y_{lm}(\theta, \phi) \\ &= R_{nl}(r)\sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}}P_{lm}(\cos\theta)e^{im\phi} \end{aligned} \quad (107)$$

Now spatial inversion $\mathbf{r} \rightarrow -\mathbf{r}$ is equivalent to

$$\begin{aligned} r &\rightarrow r \\ \theta &\rightarrow \pi - \theta \\ \phi &\rightarrow \pi + \phi \end{aligned} \quad (108)$$

See figure 3.1 of Perkins (ref. 25). Thus,

$$\begin{aligned} e^{im\phi} &\rightarrow e^{im(\pi+\phi)} = (-1)^m e^{im\phi} \\ P_{lm}(\cos\theta) &\rightarrow P_{lm}(\cos(\pi-\theta)) = (-1)^{l+m} P_{lm}(\cos\theta) \end{aligned} \quad (109)$$

or

$$Y_{lm}(\theta, \phi) \rightarrow Y_{lm}(\pi - \theta, \pi + \phi) = (-1)^l Y_{lm}(\theta, \phi) \quad (110)$$

Therefore, the *spherical harmonics have parity* $(-1)^l$.

Now the selection rules for photon transitions between atomic states are

$$\begin{aligned} \Delta m &= \pm 1, 0 \\ \Delta l &= \pm 1 \end{aligned} \quad (111)$$

Thus, when an atomic transition occurs, the parity of the energy levels must change, which means that the *photon must carry away parity of -1* so that the parity of the whole system is conserved (refs. 24 and 25). Thus, if the photon has an intrinsic parity, it makes sense that other particles can be assigned an intrinsic parity.

There is a certain parallel between parity and angular momentum (ref. 2). Just as angular momentum comes in two varieties, both orbital and intrinsic, so too does parity. We have seen that the orbital or relative parity is $(-1)^l$ due to the spherical harmonics. Particles also have intrinsic spin and intrinsic parity.

In quantum field theory one can show that (ref. 23) *bosons have the same intrinsic parity as their antiparticles, whereas fermions have opposite parity to their antiparticles.*

The reference for the rest of this subsection is by Das and Ferbel (ref. 23). Consider parity conservation in the following decay (ref. 23):

$$A \rightarrow B + C \quad (112)$$

Let J be the spin of the decaying particle A . If the two particles, B and C , are spinless, then their relative angular momentum (l) must equal the spin of A ,

$$l = J \quad (113)$$

Conservation of parity implies (ref. 23)

$$\eta_A = \eta_B \eta_C (-1)^l = \eta_B \eta_C (-1)^J \quad (114)$$

However, if $J = 0$, then

$$\eta_A = \eta_B \eta_C \quad (115)$$

In this case, the allowed decays are

$$\begin{aligned} 0^+ &\rightarrow 0^+ + 0^+ \\ 0^+ &\rightarrow 0^- + 0^- \\ 0^- &\rightarrow 0^+ + 0^- \end{aligned} \quad (116)$$

where the notation convention is J^P . If parity is conserved, the following reactions cannot take place (ref. 23):

$$\begin{aligned} 0^+ &\neq 0^+ + 0^- \\ 0^- &\neq 0^+ + 0^+ \\ 0^- &\neq 0^- + 0^- \end{aligned} \quad (117)$$

8.5.9. Parity of Deuteron

The proton and neutron are both spin $1/2$ particles, and in an s-state can form either a $J = 0$ or $J = 1$ particle. The deuteron spin is $J = 1$, indicating that the proton and neutron both have spins up in the deuteron.

The proton and neutron both have parity $\eta = +1$. The parity of the deuteron is

$$\eta_d = \eta_n \eta_p (-1)^l \quad (118)$$

Now in the deuteron, the proton and neutron are primarily in a relative s-state ($l = 0$), so that

$$\eta_d = \eta_n \eta_p (-1)^l = (+1)(+1)(-1)^0 = +1 \quad (119)$$

The intrinsic parity of the deuteron is $+1$.

8.5.10. Parity of π^-

References 12 and 23 are excellent references for this section. In this section we shall see how it is determined that the π^- has negative parity. Consider the absorption of a very low energy π^- on deuterium,

$$\pi^- + d \rightarrow n + n \quad (120)$$

It is known (ref. 12) that the pion slows down in the target and is finally captured around a deuteron, forming a pionic atom. With the emission of photons, the pion rapidly goes down to the lowest energy level, which is an s-state ($l = 0$) from which the above reaction occurs; that is, this final s-state is unstable and the pion reacts with the deuteron. Thus, the initial relative angular momentum l_i between the π^- and d is $l_i = 0$.

Conservation of parity requires that

$$\eta_\pi \eta_d (-1)^{l_i} = \eta_n \eta_n (-1)^{l_f} = (-1)^{l_f} \quad (121)$$

where the far right-hand side occurs because we have parities of two identical particles multiplied together ($\eta_n \eta_n$), which always gives $+1$. Now the deuteron parity is $\eta_d = +1$, and also $l_i = 0$ so that

$$\eta_\pi = (-1)^{l_f} \quad (122)$$

where l_f is the relative angular momentum of the final state of the two neutrons. These are two identical fermions, and thus their total wave function must be antisymmetric.

If the two neutrons have opposite spin, then the spin wave function is antisymmetric; therefore, the orbital wave function must be symmetric, leading to $l_f = \text{even}$. However,

in the $\pi^- + d \rightarrow n + n$ reaction, the initial angular momentum is 1 (π^- has spin 0, d has spin 1, and as we saw $l_i = 0$). Thus, angular momentum conservation would be violated. Thus, the two neutrons must both have the same spin.

In this case, the spin wave function is symmetric and therefore the orbital wave function must be antisymmetric leading to $l_f = \text{odd}$. Thus, we finally have $\eta_\pi = -1$.

8.5.11. Weak Interactions Violate Parity

Some good references for this section are Das (ref. 23) and Griffiths (ref. 2). A big question occurred in the early 1950s concerning the *tau-theta puzzle*. Two strange mesons, at the time called θ and τ , seemed to be identical in every respect (same mass, lifetime, charge, spin and so on) except that their weak decays were quite different, namely

$$\theta^+ \rightarrow \pi^+ + \pi^0 \quad (123)$$

$$\begin{aligned} \tau^+ &\rightarrow \pi^+ + \pi^0 + \pi^0 \\ &\rightarrow \pi^+ + \pi^+ + \pi^- \end{aligned} \quad (124)$$

Both τ and θ have spin zero, and from angular momentum conservation, the total final angular momentum must also be 0. Now the pions all have spin 0 and thus, from conservation of angular momentum, the total final relative value of orbital angular momentum must be $l_f = 0$. Consequently, the final orbital parity is +1. Now all the pions have negative parity, and the final total parity must be +1 for the θ decay and -1 for the τ decay. Assuming conservation of parity implies that the initial, or intrinsic parity of θ was +1 and for τ it was -1 . The great puzzle was that two seemingly identical particles should carry opposite parity. This puzzle was resolved by the suggestion of Lee and Yang (in 1956) that weak interactions don't conserve parity and that the θ and τ are actually the *same* particles, which is now called the positive kaon K^+ .

8.5.12. Charge conjugation parity

Some good references for this section are Das (ref. 23) and Griffiths (ref. 2). Classical electrodynamics is invariant under a change in the sign of the charge (ref. 20). Charge conjugation is a generalization of “changing the sign of charge.” Charge conjugation is the operation which *converts a particle to an antiparticle* and vice versa. As Griffiths (ref. 2) points out, the word charge conjugation is something of a misnomer because it is also applicable to neutral particles.

Just as ordinary parity is the operation that changes $\mathbf{r} \rightarrow -\mathbf{r}$ or $P\psi(\mathbf{r}) \equiv \psi(-\mathbf{r})$, the charge conjugation parity operator C changes particles to antiparticles,

$$C|p\rangle \equiv |\bar{p}\rangle \quad (125)$$

In other words, the charge conjugation operator *flips the sign of all quantum numbers* that pertain to a particle. As with ordinary parity, define an eigenvalue equation

$$C|p\rangle \equiv \eta_c|p\rangle \quad (126)$$

where η_c is the *eigenvalue or charge conjugation parity quantum number*. Now let C act again as in

$$\begin{aligned} C^2|p\rangle &= C|\bar{p}\rangle = |\bar{p}\rangle \\ &= \eta_c C|\bar{p}\rangle = \eta_c^2|p\rangle \end{aligned} \quad (127)$$

Thus, $\eta_c^2 = 1$ or

$$\eta_c = \pm 1 \quad (128)$$

(Because $C^2 = 1$, we say that C is idempotent.) Thus, particles and antiparticles will be eigenstates of the charge conjugation parity operator with charge conjugation quantum numbers of ± 1 .

A charge conjugation parity quantum number cannot be associated with charged particles (ref. 25) because, for example,

$$C|\pi^+\rangle \equiv |\pi^-\rangle$$

and thus

$$C|\pi^+\rangle \neq \pm|\pi^+\rangle$$

Thus only neutral particles will have a well-defined charge conjugation parity quantum number.

One can show (refs. 2 and 25) that for a system consisting of a spin-1/2 particle and its antiparticle, with relative angular momentum L and *total* spin S , then the eigenvalues are given by

$$\eta_c = (-1)^{L+S} \quad (129)$$

Thus mesons (made of a quark-antiquark pair) have their C eigenvalue given by this equation.

C is conserved in strong and EM interactions but is violated in weak interactions.

Like ordinary parity, charge conjugation parity is a multiplicative quantum number. Consider the EM decay of the pion into two photons

$$\begin{aligned} \pi^0 &\rightarrow \gamma + \gamma \\ \eta_c : \quad (+1) &\rightarrow (-1) \times (-1) \end{aligned} \quad (130)$$

where C is conserved. However, decay into three photons is forbidden by C conservation, although decay into four photons is allowed.

8.5.13. Conservation of CP and CPT

A good reference for this subsection is reference 36. All interactions obey CP conservation except that CP violation has been observed in the weak decays of K^0 mesons. Thus, if CP is violated in K^0 weak decays, it means that T -parity is also violated.

The time reversal operator T is antiunitary and therefore does not have a quantum number associated with it (ref. 12). Thus, the time parity quantum number T_p does not exist and therefore a corresponding C_pPT_p quantum number does not exist either. Nevertheless, it is still possible to analyze T and CPT invariance in particle reactions.

8.5.14. Conservation of G Parity

Griffiths (ref. 2) has a beautifully clear explanation of G parity. References 14 and 19 are other excellent references for this section. G parity, which is another multiplicative quantum number, is defined as

$$\eta_G = \eta_C(-1)^I \quad (131)$$

where I = isospin of multiplet. G parity is conserved whenever C and I are conserved (ref. 14), which means that G parity is conserved in strong interactions but violated in EM and weak interactions.

Thus, the pion multiplet has $I = 1$ and the π^0 has $\eta_C = +1$, giving $\eta_G = (+1)(-1)^1 = -1$. Similarly, the ρ meson has $\eta_G = +1$ and the ω meson has $\eta_G = -1$.

8.5. 15. Leptons Versus Hadrons

Leptons have baryon number $A = 0$ and so S, Y, I, I_z are not defined for leptons. They are only quantum numbers relevant for baryons. See tables 3 and 4.

9. Selection Rules

References 17, 23, 29, and 31 are best for selection rules. References 30, 32, 34, 36, and 37 are also other good references.

In *Electromagnetic* interactions, even though total isospin is violated, it still obeys a selection rule (refs. 23 and 29).

$$\Delta I = 0, \pm 1 \quad (132)$$

Many of the other quantum numbers listed in table 2 are violated in *weak* interactions but also still obey certain selection rules. It is convenient to divide up the weak interactions into the three categories of Leptonic, Semileptonic, and Nonleptonic. Leptonic processes involve *only* leptons, while Nonleptonic processes do not involve *any* leptons, but only hadrons. Semileptonic processes involve both hadrons and leptons. The selection rules for these weak interaction processes are discussed in references 17 and 29.

10. Concluding Remarks

We have presented a detailed study for all the issues related to the inclusion of exclusive reaction cross sections for pions and nucleons in Langley transport methods with particular reference to the extension of HZETRN code for mesons. This extension would be extremely useful for space missions with particular reference to radiation protection and shielding from hazards from cosmic radiations.

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